

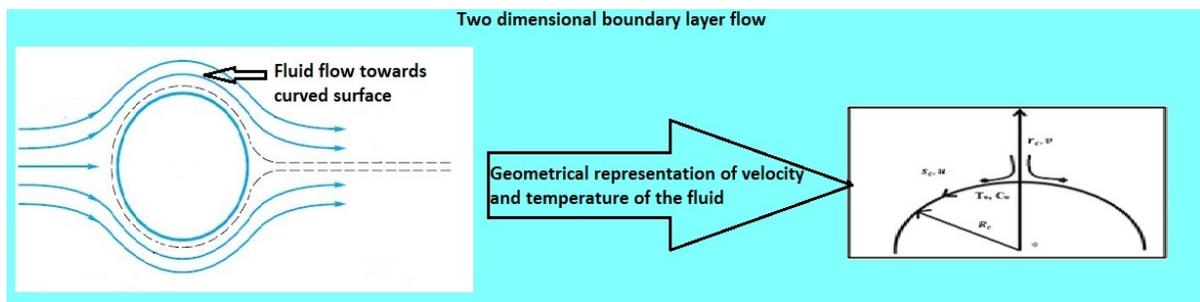
Numerical study due to mixed convection nanofluid flow with the effect of velocity slip and thermal conductivity across curved stretching surface

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ABSTRACT



The current research investigates mixed convection across curved stretching surface. This analysis takes into account the effect of velocity slip as well as thermal conductivity. The boundary layer problem is expressed as a mathematical system of equations. Equations in a non-dimensional form are derived by applying an appropriate similarity transformation. Matlab is employed to compute the numerical solutions of the highly nonlinear system of ordinary differential equations. For various values of relevant parameters, substantial variations in the velocity, temperature, and concentration profiles were found. Graphs and tables are used to illustrate the results. It has been shown that due to the rising value of curvature parameter the skin friction coefficient drops.

Keywords: Mixed convection, Velocity Slip, Thermal conductivity, Curved stretching surface

INTRODUCTION

The beginning of the background of boundary layer flow has been discussed in recent years. researchers and scientists have focused on various elements of heat transfer and transformation over a stretched surface. The viscous fluid flow toward the boundary layer caused by an incessantly moving or stretching sheet has several engineering and technological applications. Manufacturing of paper, vehicles, medical device manufacture, and other industrial activities are examples of such processes. The term "mixed convection flow" refers to a flow that combines forced and free convection. Mixed convection flow is the combination of forced and free convection flows. This kind of flow is found in variety of transport processes in both engineering and nature, such

as flows in the ocean, and atmospheric flows, nuclear reactors, electronic devices cooled by fans, etc. Choi et al.¹ was the first to investigate the heat transfer of nanofluid flow due to convection. Pandey et al.² addressed the MHD convective flow of nanofluid across the curved surface with the effect of different physical quantities. Combined impact of chemical reactions and buoyancy forces was explored by Revathi et al.³. Ahmad et al.⁴ discussed the two-dimensional viscous fluid flow with the combined effect of magnetic field and mixed convection due to curved stretching surface. The steady fluid flow towards the stagnation point, as well as the joule heating impact, was studied by Zhang et al.⁵. Ahmad et al.⁶ obtained the dual solution of boundary layer flow problem numerically for Sisko fluid, the impact of thermal radiation due to higher heat generation and chemical reaction on the flow, also considered in this study. Acharya et al.⁷ discussed the advancement in increasing value of heat transfer coefficient due to first and second-order velocity slip effect, on MHD fluid flow. Naveed et al.⁸ examined the magnetohydrodynamic incompressible fluid flow and heat transfer by applying a curvilinear coordinate system. Ibrahim et al.⁹ investigated the higher-order slip flow of nanofluid

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and the impact of thermo-diffusion was also explored in this work. Wahid et al.¹⁰ developed a viscous fluid flow model along with the radiation effect across the curved surface. Brownian motion and the thermophoresis effect were found by Awais et al.¹¹ on heat transfer through a vertical stretched sheet. Due to its wide practical use, this work drew a lot of interest in the field of heat and mass transfer. Khan et al.¹² used similarity transformation to find a numerical solution of the nanofluid flow issue. The flow of Casson fluid with the heat source/sink effect owing to a stretched cylindrical surface was investigated by Song et al.¹³. In order to predict the boundary layer flow pattern, many studies included the different variable properties of the fluid. The influence of first order velocity slip along with consequences of higher value of heat generation and absorption constants on the flow of nanofluid across curved surface was discussed by Muhammad et al.¹⁴. Khan et al.¹⁵ find out the numerical solution as well as analytical solution of the electrically conducting fluid flow problem due to porous effect on the curved surface area. Amanulla et al.¹⁶ studied the consequences of various slip conditions on MHD viscous fluid flow in an isothermal sphere domain in the porous medium. The impact of radiation parameter on two-dimensional Casson fluid flow was examined by Zhou et al.¹⁷. Rosca et al.¹⁸ studied the modelling of unsteady flow of nanofluid along both cases stretching and shrinking surfaces with the effect of chemical reactions. And numerical solutions of the problem were obtained with curvilinear coordinates. Afterwards, Ahmed et al.¹⁹ inspected the flow pattern across exponentially stretching curved surface to provide the impacts of different physical aspects such as thermal conductivity, permeability on Williamson nanofluid flow. Imtiaz et al.²⁰ studied the mutual effect of Joule heating and thermal conductivity on Casson fluid flow, the rate of heat and mass transfer were also inspected for these parameter in this study. Xiong et al.²¹ discussed the velocity slip effect on velocity and temperature distribution. many studies²²⁻²³ have examined the velocity and thermal slip on convective fluid flow. Kumar et al.²⁴ used the heat source and sink effect on the flow of Carreau nanofluid past an exponentially stretching sheet. Abbas et al.²⁵ discussed the viscous boundary layer flow of Casson fluid due to magnetic field present towards the vertical direction of the fluid flow. various investigations have been done by²⁶⁻²⁸ in the field of curved surface. Khan et al.²⁹ considered the mixed convective Jeffery fluid flow model to investigate the influence of sundry flow variables on the velocity at curved surface. Imtiaz et al.³⁰ developed the viscous fluid model, and also enlightened the impacts of homogeneous-heterogeneous reactions on change in temperature distribution of the fluid. Abbas et al.³¹ studied that thermal radiation and natural convection both are the major factors, that effects the flow of hydromagnetic fluid. Liu et al.³² presented an unsteady flow of nanofluid towards the stagnation point.

MATHEMATICAL FORMULATION

The current study focused on the flow of mixed convection nanofluids towards a curvilinear stretching surface. The velocity slip boundary condition with the effect of thermal radiation was used to conduct the study. The curvilinear coordinates system (s_c, r_c) was utilized for solving the two-dimensional fluid flow problem. s_c Coordinate measured along a curved surface whereas

the coordinate r_c is normal to the curved surface. The Radius of curvature of the Curved surface assuming R_c, T_w and C_w is the surface temperature and concentration of the surface. T_∞ is ambient temperature i.e. Temperature at far from the surface and C_∞ is the concentration far from the surface area.

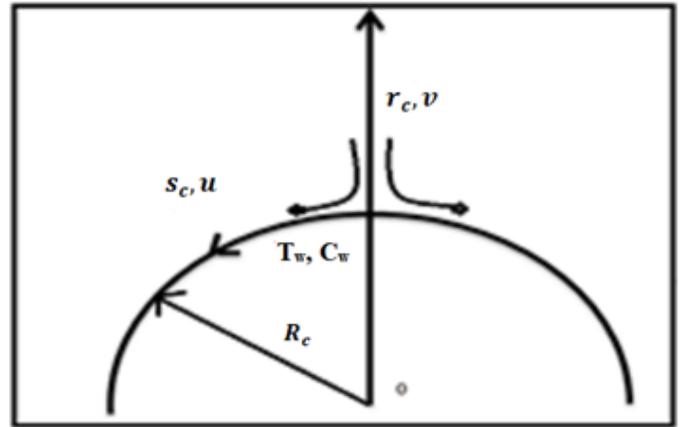


Figure 1 Physical representation of fluid flow

The mixed convective flow problem of a nanofluid along with appropriate boundary conditions govern as [33-34]

$$\frac{\partial}{\partial r_c} \{ (r_c + R_c) v \} + R_c \frac{\partial u}{\partial s_c} = 0 \tag{1}$$

$$\frac{1}{\rho_{nf}} \frac{\partial p}{\partial r_c} - \frac{u^2}{r_c + R_c} = 0 \tag{2}$$

$$v \frac{\partial u}{\partial r} = \frac{1}{\rho_{nf}} \frac{R_c}{r_c + R_c} \frac{\partial p}{\partial s_c} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial u}{\partial r_c} - \frac{u}{(r_c + R_c)^2} \right) - \frac{vu}{r_c + R_c} + \frac{R_c u}{r_c + R_c} \frac{\partial u}{\partial s_c} + \frac{g}{\rho_{nf}} [(\rho\beta_t)_{nf}(T - T_\infty) + (\rho\beta_c)_{nf}(C - C_\infty)] \tag{3}$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r_c} + \frac{R_c u}{r_c + R_c} \frac{\partial T}{\partial s_c} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial T}{\partial r_c} \right) \tag{4}$$

$$v \frac{\partial C}{\partial r_c} + \frac{R_c u}{r_c + R_c} \frac{\partial C}{\partial s_c} = D_m \left(\frac{\partial^2 C}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial C}{\partial r_c} \right) \tag{5}$$

With boundary condition

$$u = U_w + \delta^* \frac{\partial u}{\partial r_c} \text{ as } U_w = a s_c, \text{ where } a \text{ is a constant}$$

$$u = a s_c + \delta^* \frac{\partial u}{\partial r_c}, v = 0, T = T_w, C = C_w \text{ at } r_c = 0 \tag{6}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \frac{\partial u}{\partial r_c} \rightarrow 0 \text{ as } r_c \rightarrow \infty \tag{7}$$

u and v are nanofluid velocity along curvilinear axis s and r_c direction, $(\rho C_p)_{nf}$ is the specific heat of nanofluid D_m is Molecular diffusivity, δ^* is slip coefficient, ρ_{nf} is density of nanofluid, k_{nf} is Thermal conductivity of the fluid, μ_{nf} is dynamic viscosity of the nanofluid, R_c is Radius of curved surface.

NUMERICAL SOLUTION

Following similarity transformation implemented for obtaining non-dimensional equations of the equations. Singh et al.³⁵

$$\xi = \left(\frac{a}{v_f} \right)^{1/2} r_c, u = a s_c g'(\xi), p = \rho_f (a s_c)^2 P(\xi)$$

$$v = - \frac{R_c}{r_c + R_c} \sqrt{a v_f} g(\xi), T = T_\infty + (T_w - T_\infty) \theta(\xi)$$

$$C = C_\infty + (C_w - C_\infty) \phi(\xi)$$

Eq. 1 is satisfied by the above similarity transformation and the reduced equations are: Bhattacharya et al.³⁶

$$\frac{\rho_{nf}}{\rho_f} \frac{\partial P}{\partial \xi} = \frac{g'^2}{\xi+K} \tag{8}$$

$$\frac{\rho_f}{\rho_{nf}} \frac{2K}{\xi+K} P = \frac{\nu_{nf}}{\nu_f} \left(g''' - \frac{1}{(\xi+K)^2} g' + \frac{1}{(\xi+K)} g'' - \lambda g' \right) - \frac{K}{(\xi+K)} (g')^2 + \frac{K}{(\xi+K)} g g'' + \frac{K}{(\xi+K)^2} g g' + \lambda \theta + K_2 \phi \tag{9}$$

$$\frac{1}{Pr} \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} \left(\theta'' + \frac{1}{(\xi+K)} \theta' \right) + \frac{K}{(\xi+K)} g \theta' = 0 \tag{10}$$

$$\phi'' + \frac{K}{(\xi+K)} Sc g \phi' + \frac{\phi'}{(\xi+K)} = 0 \tag{11}$$

With boundary condition

$$g'(0) = 1 - \delta g''(0), g(0) = 0, \theta(0) = 1, \phi(0) = 1 \text{ at } \xi = 0 \tag{12}$$

$$g' \rightarrow 0, g'' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \xi \rightarrow \infty \tag{13}$$

To eliminate the pressure term from equation number (8) & (9) we differentiate equation no (9) w.r.t. ξ then put the value of $\frac{\partial P}{\partial \xi}$ from Eq. no. (8) We get

$$g^{iv} + \frac{2}{\xi+K} g''' - \frac{1}{(\xi+K)^2} g'' + \frac{1}{(\xi+K)^3} g' - \lambda g' + \frac{\nu_{nf}}{\nu_f} \left\{ \frac{K}{(\xi+K)} (g g''' - g' g'') + \frac{K}{(\xi+K)^2} (g g'' - g'^2) - \frac{K}{(\xi+K)^3} g g' \right\} + \lambda \left[\theta' + \frac{\theta}{(\xi+K)} + k_2 \left(\phi' + \frac{\phi}{(\xi+K)} \right) \right] = 0 \tag{14}$$

Where $\delta = \lambda \sqrt{\frac{a}{\nu_f}}$ is slip parameter, $\lambda = \frac{\rho \beta_t (T - T_\infty) / \nu^2}{(sU_w)^2 / \nu^2}$ mixed convection parameter, $K = R_c \sqrt{\frac{a}{\nu_f}}$ is Curvature parameter, $K_2 = \frac{\beta_c (T - T_\infty)}{\beta_t (C - C_\infty)}$ buoyancy ratio parameter, $Sc = \frac{\nu_f}{D_m}$ is schmidt number, $Pr = \frac{\nu_f (\rho C_p)_{nf}}{k_{nf}}$ is prandtl number.

The physical quantities of interest at a curved surface, coefficient of skin friction C_f , local Nusselt number Nu , and Sherwood number Sh are determined as follows:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, Nu = \frac{sq_w}{k_{nf} (T_w - T_\infty)}, Sh = \frac{s j_w}{D_m (C_w - C_\infty)} \text{ Where}$$

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial r_c} - \frac{u}{r_c + R_c} \right)_{r_c=0} \text{ is shear stress at wall, } q_w = -k_{nf} \left(\frac{\partial T}{\partial r_c} \right)_{r_c=0} \text{ is surface heat flux,}$$

$$j_w = -D_m \left(\frac{\partial C}{\partial r_c} \right)_{r_c=0} \text{ Surface mass flux in the direction of } s_c \text{ axis.}$$

RESULTS AND DISCUSSION

To solve the nonlinear differential equations (10) (11) and (14) with the associated boundary conditions (12) and (13), MATLAB software is used. Velocity distribution, temperature distribution along with concentration profile demonstrated for related constant parameters. The certain flow parameters retained fixed for the whole study $K = 10, Pr = 1, Sc = 1.0, \lambda = 0.3, \delta = 0.5, K_2 = 0.3$.

Figure 2 demonstrates the change in velocity, owing to slip parameter δ . The velocity field and dynamic viscosity has converse relationship, and has a comparable impact. Because the slip parameter raises the value of dynamic viscosity, consequently the

fluid motion decreases significantly, and this phenomenon causes the drop in the velocity profile. Figure 3 is plotted to highlight the variations in velocity distribution due to changing value of curvature parameter K . The bent of the curved extending surface aids fluid flow across it. The outcome of shown figure is that the velocity improves as the curvature parameter K rises gradually.

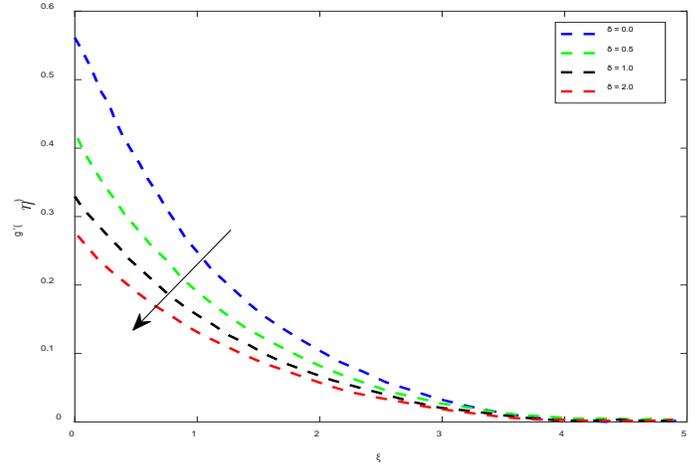


Figure 2 velocity profile versus δ

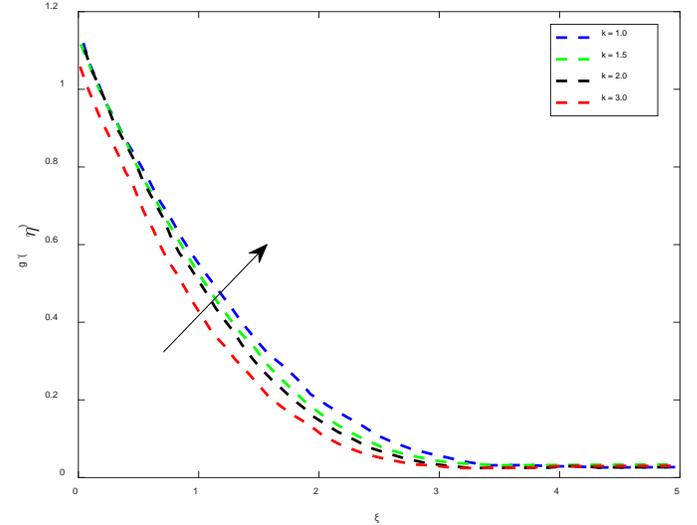


Figure 3 velocity profile versus K

Figure 4 elucidates the influence for different values of slip parameter δ on temperature distribution. It can be easily observed through the figure that due to slight increment in the velocity slip δ , the friction at the boundary layer increases, allowing more heat to be transmitted to the fluid, causing the temperature to rise significantly. Figure 5 illustrates the behavior of the temperature distribution in the fluid as the curvature parameter K changes. The rising value of the parameter K causes the reduction in temperature. Figure 6 demonstrates the variation in velocity distribution for the different values of λ . Due to rising value of mixed convection parameter λ , the significant progress observed in Velocity profile. A higher value of λ causes the buoyancy forces to rise, resulting velocity distribution grows due to this phenomenon. Figure 7

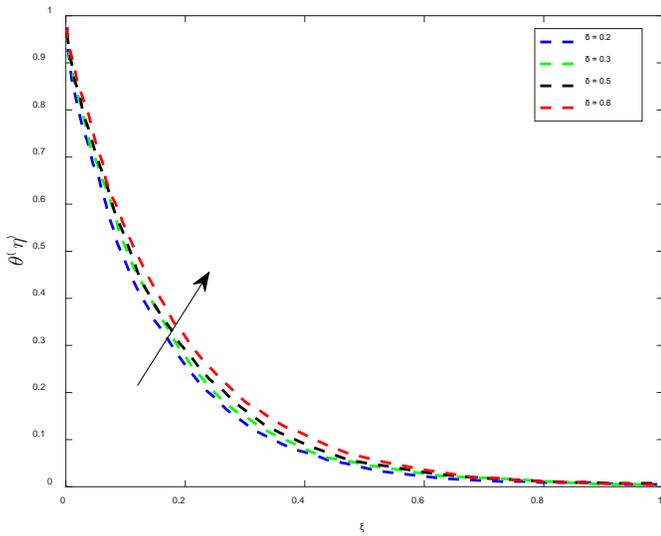


Figure 4 temperature profile versus δ

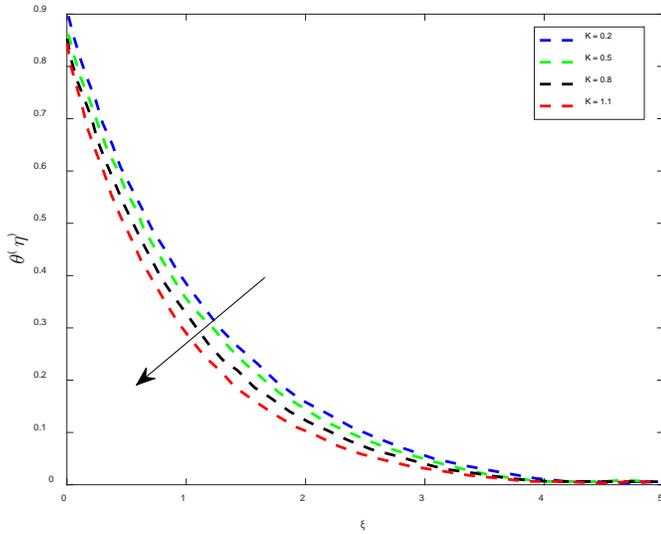


Figure 5 temperature profile versus K

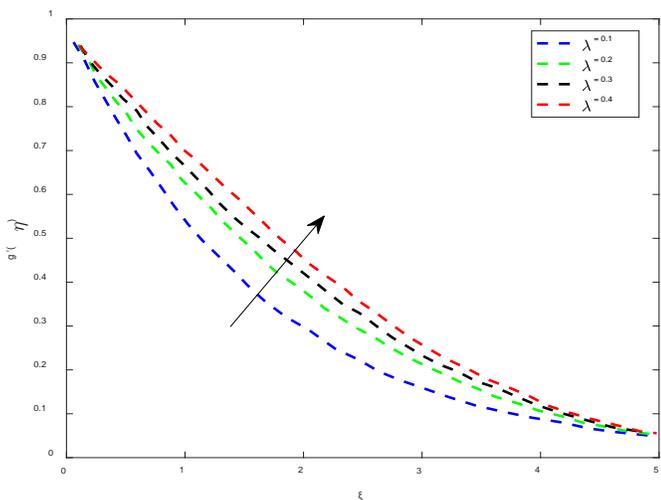


Figure 6 Velocity profile versus λ

exhibits the variation in skin friction coefficient for changing value of curvature parameter K . It has been noticed that with the larger value of the curvature parameter K , The skin friction coefficient appears to decrease.

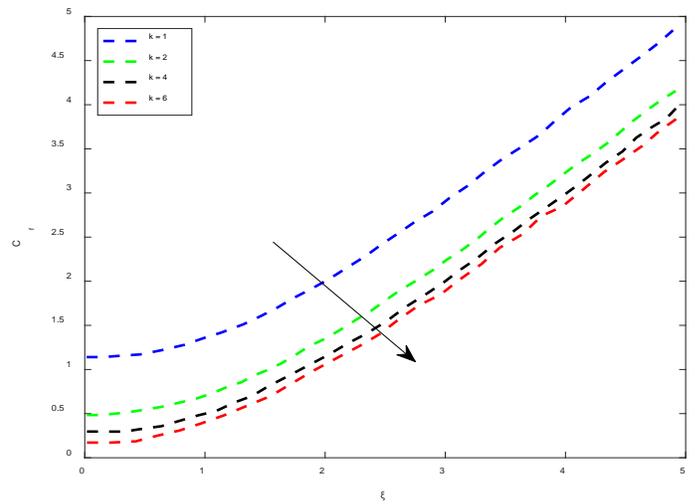


Figure 7 Skin friction coefficient versus K

The variations in the concentration profile for the mixed convection parameter are depicted in Figure 8. The graph shows a drop in concentration profile due to the effect of buoyancy force which enhances the pressure gradient, due to this reason concentration profile is decreases. Irfan et al.³⁷

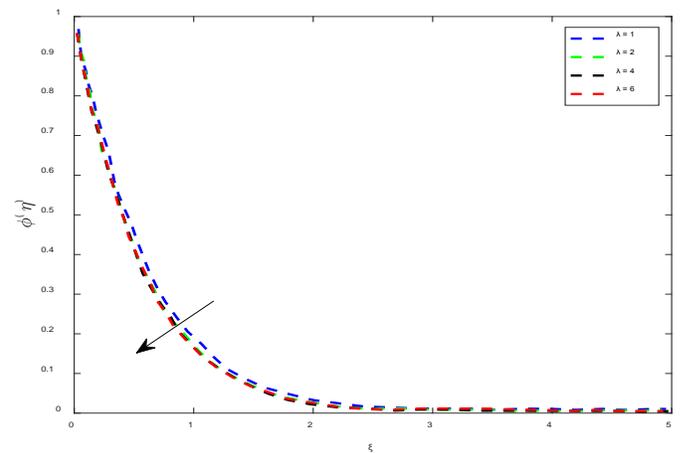


Figure 8 Concentration profile versus λ

Figure 9 demonstrates the features of concentration of fluid versus Schmidt number. As Sc increases, the momentum dissipation rate improves, the concentration of the fluid drops. For varying values of K , Pr , and Sc . **Table 1** shows the Local Nusselt number and Sherwood number. For varying Pr and Sc , the Local Nusselt number grows as the curvature parameter grows. The findings support earlier research and show that the findings are in good accord. In the below table we can see that the nusselt number

for $Pr = 1.4$ and $Pr = 1.8$ are identical to the result of Ahmed et al.¹⁹ upto five decimal places.

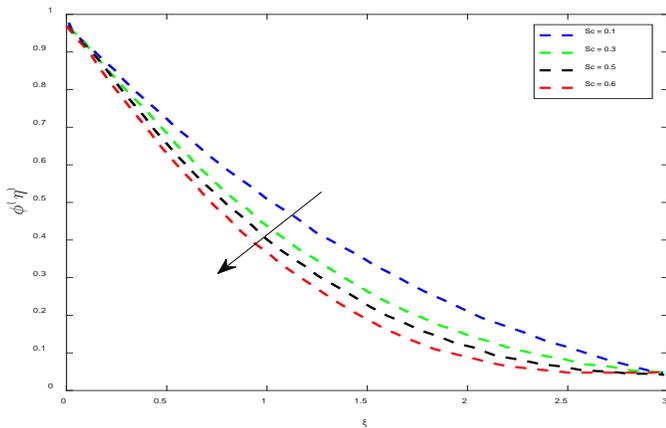


Figure 9 Concentration profile versus Sc

Table 1: Comparison of Local Nusselt number and Sherwood number for the K, Pr, Sc

K	Pr	Sc	$-\theta'(0)$	[19]et al.	$-\phi'(0)$	[19]et al.
1	1.0	0.6	1.081935728		1.087855437	1.08785273
3	1.3	0.8	1.082142173		1.085473158	
5	1.4	1.0	1.082527168	1.08252	1.140732428	1.14073275
6	1.5	1.2	1.084471589	1.084473678	1.175715642	

CONCLUSION

The mixed convective nanofluid flow was studied in the current work. This study not only highlighted the thermal radiation and velocity slip condition impacts on the heat transfer but also investigated the different aspects of concentration of the fluid for the related parameters. The numerical outcomes were explored for the various pertinent parameters. The physical elucidation of skin friction coefficient, local Nusselt number, and Sherwood number for the concerned variables are displayed in the form of graphical as well as tabular form.

The following are the key findings:

- Velocity distribution decays for the rising value of slip parameter δ but converse behavior observed for curvature parameter K.
- Temperature profile rises for larger value of slip parameter δ but temperature decreases for the higher value of curvature parameter K.
- Velocity profile increases for the enhancing value of λ .
- Reduction in Skin friction coefficient seen with the larger value of curvature parameter.

CONFLICT OF INTEREST

Authors declared no conflict of interest of any kind for publication of this article.

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